The Invisible Link Between Mathematics and Theology

Ladislav Kvasz

If we compare the mathematics of antiquity with that of the seventeenth century, we find differences in a whole range of aspects. For the ancients, notions like infinity, chance, space, or motion fell outside mathematics, while in the seventeenth century new mathematical theories about these notions appeared. I believe that this fundamental change can be ascribed to the influence of theology. For the ancients, ontology and epistemology were in unity. They considered the world to be as it appeared to them; the phenomena as infinity or chance, which appeared to them as ambiguous, they held to be really so. For modern humanity, ontology and epistemology differ in a fundamental way. The being of the world is determined by the omniscient God, therefore it is perfect, while our knowledge of the world is determined by our finite capacities, and therefore it is ambiguous. It is this gap between ontology and epistemology, which makes the mathematicization of notions such as infinity or chance, despite their apparent ambiguity, possible.

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In the history of mathematics, we can find several topics that reveal a direct connection between mathematics and theology. Perhaps the most famous of them is set theory, connected with the transition from the concept of the potential infinity to that of the actual infinity. In the works of Bernard Bolzano and Georg Cantor, the founders of set theory, we find theological influences, the analysis of which plays an important role in the understanding of the history of that theory.[1]

Another topic revealing the encounter of mathematics and theology is mathematical logic. Gottlob Frege and Bertrand Russell mark the end of a long tradition focused on critical examination of the various proofs of God's existence, in the course of which many of the principles of modern logic were discovered.[2] To illustrate this, it is sufficient to mention Kant's thesis, according to which existence is not a real predicate. Kant formulated this thesis in his criticism of Anselm's ontological proof of God's existence (as existence is not a real predicate, from the premise that all positive predicates apply to God, his existence does not follow). In mathematical logic, Kant's thesis is one of the principles of the syntax of predicate calculus. In accordance with this principle, existence is formalized by using quantifiers and not predicates. Nevertheless, besides such direct connections between mathematics and theology we also can find a hidden but, in my view, an even more important influence of theology on mathematics. This hidden influence concerns the boundary, discriminating the phenomena open to mathematical description from those which defy mathematical description.

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In the first part of this article, I present five examples from the history of mathematics that illustrate the deep changes which occurred in this discipline between the late antiquity and the early modern era. Each of these examples, taken separately, is well known in the history of mathematics, but by putting them together a *common pattern of change* seems to appear. In each of the five cases, a phenomenon considered by the ancients to defy mathematical description

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was mathematicized. The thesis of this article is that monotheistic theology with its idea of the omniscient and omnipotent God, who created the world, indirectly influenced the process of this mathematicization. In *separating ontology from epistemology*, monotheistic theology opened the possibility to explain all of the ambiguity connected to these phenomena as a result of human finitude and so to understand the phenomena themselves as unambiguous, and thus accessible to mathematical description.

Shifts of the Boundaries of Mathematicization in Early Modern Era

If we want to analyze the implicit, indirect way in which monotheistic theology influenced the development of western mathematics, it seems appropriate to compare the mathematics of late antiquity with that of the sixteenth and seventeenth centuries. In this later period, western mathematics, after long centuries of decline and stagnation, finally reached an intellectual level comparable with the late Hellenistic era, characterized by the work of Archimedes and Apollonius. Comparing the mathematics of these two epochs, we find a surprising fact. The mathematics of the sixteenth and seventeenth centuries was not simply a revival of the ancient tradition. It differed from the mathematics of the sixteenth and seventeenth centuries in a whole range of aspects, which, in my view, can be ascribed to the influence of monotheistic theology on mathematics. In order to get a better understanding of these aspects, I will concentrate on five notions that underwent radical changes. These are the notions of infinity, chance, the unknown, space, and motion.

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1. *Apeiron*—infinity. What we refer to today as infinite was in antiquity subsumed under the notion of *apeiron* (απειρον). Nevertheless, compared with our modern notion of infinity, the notion of *apeiron* had a much broader meaning. It applied not only to that which was infinite, but also to everything that had no boundary (i.e. no *peras*), that was indefinite, vague, or blurred. According to ancient scholars, *apeiron* was something lacking boundaries, lacking determination, and therefore uncertain. Mathematical study of *apeiron* was impossible, mathematics being the science of the determined, definite, and certain knowledge. That which had no *peras* could not be studied using the clear and sharp notions of mathematics.

Modern mathematics, in contrast to antiquity, makes a distinction between infinite and indefinite. We consider the infinite, despite the fact that it has no end (*finis*), to be determined and unequivocal, and thus accessible to mathematical investigation. Be it an infinitely extended geometrical figure,[3] an infinitely small quantity[4] or an infinite set,[5] we consider them as belonging to mathematics. The ancient notion of *apeiron* was thus divided into two notions: the notion of the *infinite* in a narrow sense, which became a part of mathematics, and the notion of the *indefinite*, which, as previously, has no place in mathematics.

2. *Tyché*—randomness. Another difference between the ancient and modern mathematics appears in the understanding of randomness (*tyché*–τυχη). The notion of *tyché*, similarly to that of *apeiron*, had a much broader meaning than our modern notion of randomness; besides random events, it also designated chance, luck, and fate in general. Therefore, it was not accessible to mathematical investigation. *Tyché* belonged in the competence of an oracle than that of mathematics. For ordinary people, their personal fate remained hidden. From the sixteenth century onward, books on gambling started to appear in mathematical literature, and during the seventeenth century, the modern probability theory developed from this tradition.[6] From the viewpoint of ancient scholars, a mathematical theory of *tyché* is just as absurd as a mathematical theory of *apeiron*. And in the case of *tyché*, the breakthrough toward modern mathematics happened along the same lines, as it did in the case of *apeiron*: the ancient notion was divided into two concepts. The first of them, the concept of randomness, became the subject of the theory of probability, while the second one, the notion of fate, remained beyond the boundaries of exact sciences.

3. *Arithmos*—unknown. The third change has to do with the birth of algebra, and especially with the notion of the unknown, which since Descartes is most often expressed by the letter x. Algebra was created in the Arabic civilization, as the name of this math-

ematical discipline indicates. The Arabic civilization is monotheistic, similar to western civilization. Thus, following my thesis about the implicit connection between monotheistic theology and modern mathematics, the birth of algebra can be placed beside the birth of the theory of probability and the mathematical theory of the infinite.

First of all, I would like to stress that the ancient mathematicians did not know the notion of the unknown in its modern, algebraic form. Of course, in antiquity, mathematicians dealt with a rich variety of practical problems, requiring them to find a certain number. The Greek mathematicians usually called this the unknown number (*arithmos*—αριϑμος). However, in solving such problems they proceeded in a synthetic way, using only the values of the known quantities given in the formulation of the problem. The unknown quantity, precisely because it was unknown, could not be used in arithmetical operations.

The basic idea of algebra is to represent the unknown by a letter and subject it to the same arithmetical operations as ordinary numbers.[7] The purpose of the algebraic symbolism is to overcome the epistemological barrier separating those quantities we know from those we do not know. Thus in algebra, we work with both the known as well as the unknown quantities, as if they were equivalent. From the point of view of ancient mathematics, this is something absurd, because if we do not know the value of a quantity, we cannot determine the outcomes of the arithmetical operations applied to it. According to the ancient understanding, what is undetermined cannot become the subject of mathematical operations.

The birth of algebra consisted of the creation of a notion of the unknown, which, despite its undetermined value, can be used as if it was fully determined. I believe that this notion can be put alongside the notions of infinity and randomness to represent the third important breakthrough of the boundaries of the sharply given, by which the world of mathematics was characterized in the ancient understanding.

4. *Kenón*—space. A change analogous to that of the notions of infinity, probability, and the unknown occurred in the transition from the ancient to modern mathematics in the understanding of space. Closest to our modern notion of space was the ancient notion of emptiness (kenón—κενον). For the ancient philosophers, with the exception of the atomists and the Epicureans, the notion of emptiness was problematic. Emptiness is where there is nothing, and so it has no specific attributes that can be studied. Not even the atomists, who accepted the existence of emptiness, were able to say much about it. Thus emptiness is surely not suitable to become a subject for mathematics. Mathematical knowledge is characterized by clear and precise notions, which is definitely not the case with the notion of emptiness.

Nevertheless, early modern science is founded on the mathematical notion of space. Newton, for instance, took the absolute space to be one of the fundamental categories of his system, and he explicitly referred to it as "mathematical" space.[8] Thus, in complete analogy to the previous three cases, a further region is mathematicized—a region that from the ancients' point of view defied any mathematicization. The new mathematical notion of space is a narrowing of the original notion of *kenón*, just as infinity was a narrowing of *apeiron* and probability was a narrowing of *tyché*. We can assert that space is three dimensional, orientable and continuous; we can hardly ascribe any of these attributes to emptiness.

From the ancient notions, which were broad and ambiguous [apeiron, tyché, kenón, and kinesis], narrow and specific parts [infinity, randomness, space, and motion] were separated, and it was only these narrower notions that were mathematicized.

5. *Kinesis*—motion. For the last example illustrating the differences between ancient and modern mathematics, let us turn to the notion of motion. The Greek notion of *kinesis* (κινησις), which designates motion, has a much broader meaning than our modern notion of motion. Besides the change of position in space, it encompasses growth, aging, and change of color. According to the ancient understanding of mathematics, *kinesis*, like *apeiron* and *tyché*, defied mathematicization. Aristotle explained why it is impossible to describe *kinesis* in mathematical terms. For an ancient scholar, a mathematical theory of *kinesis* would be the same absurdity as a mathematical theory of *apeiron* or a mathematical theory of *tyché*. The mathematicization of motion introduced by Galileo has many aspects analogous to the previous cases of mathematicization described above: the broad and general ancient notion of *kinesis* was divided into two notions. One of them was narrower, including only changes of position (i.e., local motion) and the other, a broader one, included the rest. Galileo developed a mathematical science only for the narrower notion of local motion.[9]

A Common Pattern of the Shifts of the Boundaries of Mathematicization

In the previous section, I brought together some examples documenting that, in the early modern era, some new mathematical disciplines were founded (*algebra*, *probability theory*, and *kinematics*) and a radical change occurred in our understanding of *infinity* and *space*. If we compare these five new areas of mathematical scholarship with ancient mathematics, we can see that in ancient times all of these areas were considered to defy mathematical description. The mathematicization of these areas in the sixteenth and seventeenth centuries have many common features. The first of them is that the ancient notions of *apeiron*, *tyché*, *kenón*, and *kinesis* were much broader than our modern notions of infinity, randomness, space, and motion, which became the bases of the new mathematical disciplines. Today we strictly discriminate between the infinite and the indeterminate, between randomness and fate, between emptiness and space, between motion and change. Thus from the ancient notions, which were broad and ambiguous, narrow and specific parts were separated, and it was only these narrower notions that were mathematicized.

The success of modern mathematics consisted precisely in that it has found a way to overcome the residual ambiguity of the narrow notions of infinity, probability, space, and motion.

The second common feature of the changes discussed above is that the new, narrow notions of infinity, probability, space, and motion still preserved some degree of ambiguity. However, this residual ambiguity was much weaker than the original ambiguity of the ancient notions. This weakening of ambiguity was very important, because it was precisely the ambiguity of the notions of *apeiron*, *tyché*, *kenón* and *kinesis* that led the Greek mathematicians to consider these notions as defying mathematical investigation. The success of modern mathematics consisted precisely in that it has found a way to overcome the residual ambiguity of the narrow notions of infinity, probability, space, and motion.

Now we come to the third common feature of the above-mentioned changes. Let us first take the notion of infinity. While for the ancients *apeiron* was a negative notion associated with going astray and losing the way, for the medieval scholar, the road to infinity became the road to God. God is an infinite being, but despite his infiniteness, he is absolutely perfect. As soon as the notion of infinity was applied to God, it lost its obscurity and ambiguity.[10] Theology made the notion of infinity positive, luminous, and unequivocal.[11] All ambiguity and obscurity encountered in the notion of infinity was interpreted as the consequence of human finitude and imperfection alone. Infinity itself was interpreted as an absolutely clear and sharp notion, and therefore an ideal subject of mathematical investigation.

Similarly, in the case of the notion of randomness, the consequence of God's omniscience was that the ambiguity of this notion has lost its ontological dimension and was reduced to a simple epistemological negativity. God knows the outcome of every toss of a dice in advance, and it is only due to the finiteness of the human mind that this knowledge remains hidden for us. Thus a random event, at least from God's point of view, is precisely determined and therefore suitable for mathematical description. Now it becomes comprehensible how the idea of a totally deterministic universe and the classical interpretation of probability could originate in the same mind, namely in the mind of Pierre Simon de Laplace. Determinism and randomness are two aspects of the same reality. Determinism is the ontological side and probability the epistemological side of the same world. According to Laplace, the world is absolutely deterministic, but to the human mind, it is opened only in a probabilistic way.

A similar tension between the ontological definiteness (necessary for the application of arithmetical operations) and epistemological indefiniteness is characteristic in the notion of the unknown in algebra. The unknown is unknown for us, finite beings. For God there are no unknowns at all. As soon as he looks at the formulation of an algebraic problem, he immediately sees the value of the unknown. He has no need to solve the equations, because due to his omniscience, he immediately knows the solutions. Thus, in a way similar to the case of the theory of probability, in algebra too, the *ontological ambiguity*, which prevented the Greeks from mathematicizing this area, was transformed into an *epistemological ambiguity*, having its roots in human finiteness and therefore being irrelevant with respect to mathematicization.

The case of kinematics is similar. On the ontological level, motion is perfect and absolutely determined. The fact that it appears to us as something ambiguous (for instance, we are not able to decide whether the free fall is accelerated or not) is a consequence of our imperfection alone. The notion of space is even clearer. In his *Scholium generale*, Newton characterized the absolute space as *Sensorium Dei*. Therefore the possibility of its mathematicization originates in God's perfection. To humans, only the relative, empirical space is accessible.

Monotheistic Theology as a Source of the Shifts of the Boundaries of Mathematicization

In summary, ontology and epistemology were in unity for the ancients. They considered the world to be as it appeared to them; the phenomena, which appeared to them as ambiguous and dim, they held to be really so. For modern humanity, ontology and epistemology differ in a fundamental way. The being of the world is determined by God, therefore it is unequivocal and perfect. On the other hand, our knowledge of the world is determined by the finite capacities of the human mind, and therefore it is ambiguous and dim. It is precisely this gap between epistemology and ontology, which makes possible the mathematicization of regions that are opened to us only in an ambiguous way. If all of the perceived ambiguity is attributed only to human finitude—i.e., if it is interpreted as epistemological—the mathematicization on the ontological level becomes possible.

This shows that monotheistic theology probably played a more important role in the creation of modern mathematical sciences than usually admitted. Monotheistic theology brought about a fundamental change of the general epistemological background, in that it separated ontology from epistemology. This separation led finally to the birth of modern mathematics with its notions of infinity, probability, the unknown, space, and motion. The fundamental differences between early modern mathematics and the mathematics of the Hellenistic period can be perhaps characterized as breaking of the boundaries of the unequivocally given and opening of the world of mathematics to the ambiguously given phenomena such as infinity, randomness, or motion. This is a fundamental change, perhaps the most important one since the discovery of proof and of the idea of an axiomatic system. And this fundamental change, this radical break towards modernity, is most likely linked with monotheistic theology.

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Of course, the evidence given by five episodes from the history of mathematics is far from being conclusive. My aim is not to settle the question of the role of theology in the development of science and mathematics, but to propose an indirect method of its study. As Max Weber analyzed the role of Protestant ethics in the development of modern capitalism, in a similar way, it is possible to analyze the role of monotheistic theology in the development of modern science. Monotheistic theology, like Protestant ethics, did not directly influence its development. Rather, it helped to create conditions in which the development of modern science became possible. ⁂

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Notes

[1]For the discussion of the role of theological ideas in the work of Georg Cantor (1845–1918) see chapter 6 in J. W. Dauben, *Georg Cantor: His Mathematics and Philosophy of the Infinite* (Princeton: Princeton University Press, 1979); M. Hallet, *Cantorian Set Theory and Limitation of Size* (Oxford: Clarendon Press, 1984), and chapter VIII in J. Ferreirós, *Labyrinth of Thought. A History of Set Theory and Its Role in Modern Mathematics* (Basel: Birkhauser, 1999). The theological background of the works of Bernard Bolzano (1781–1848) on set theory is discussed in P. Zlatos, *Ani matematika si nemôze byt*

istá sama sebou, Úvahy o mnozinách, nekonecne, paradoxoch a Gödelovych vetách (Even mathematics cannot be certain about itself, Essays on sets, infinity, paradoxes, and Gödel's theorems, in Slovak) (Bratislava: IRIS, 1995).

[2] See Zlatos, *Ani matematika si nemôze byt istá sama sebou*; P. Hajek, *"Goedeluv dukaz existence Boha"* (Gödel's proof of God's existence, in Czech) in J. Malina and J. Novotný, eds., *Kurt Goedel* (Brno: Universitas Masarykiana, 1996), and P. Zlatos, *Gödelov onthologicky dôkaz existencie Boha* (Gödel's onthological proof of God's existence, in Slovak) in J. Rybár, ed., *Filozofia a kognitívne vedy* (Bratislava: IRIS, 2002).

[3] One of the first infinitely extended geometrical figures was studied by Evangelista Torricelli (1608–1647) in 1646. See D. J. Struik, *A Source Book in Mathematics, 1200–1800* (Cambridge, MA: Harvard University Press, 1969), 227–32.

[4] Infinitely small quantities were used by Johannes Kepler (1571–1630) in his *Nova stereometria doliorum vinariorum*, published in Linz in 1615 and by Galileo Galilei (1564–1642) in his *Discorsi e dimonstrazioni matematiche, intorno a due nuove scienze; attenenti alla mecanica i movimenti locali*, published in Leiden 1638. See Struik, *A Source Book in Mathematics, 1200–1800*, 192–209.

[5] The notion of an infinite set was introduced by Bolzano in his *Paradoxien des Unendlichen* in 1851. See B. Bolzano, *Paradoxien des Unendlichen* (Hamburg: Felix Meiner, 1975).

[6] One of the first books on gambling was *Liber de Ludo Aleae* (Book on Games of Chance) written by Gerolamo Cardano (1501–1576) before 1565. See O. Ore, *Cardano the Gambling Scholar* (Princeton: Princeton University Press, 1953). The theory of probability arose from the works of mathematicians such as Pierre de Fermat (1601–1665), Blaise Pascal (1623–1662), Christian Huygens (1629–1695), Johann Bernoulli (1667–1748), Abraham de Moivre (1667–1754), and many others. See F. N. David, *Games, Gods and Gambling: A History of Probability and Statistical Ideas* (London: Charles Griffin, 1962).

[7] Al Khwarizmi introduced around 820 three operations with the unknown: *al gabr*, *al muquabala* and *al radd*. From the first of them, algebra got its name. See B. L. van der Waerden, *A History of Algebra: From al-Khwarizmi to Emmy Noether* (Berlin: Springer, 1985).

[8] The notion of absolute space was introduced by Isaac Newton (1643–1727) in the Sholium at the beginning of the first book of the *Philosophiae Naturalis Principia Mathematica* published in 1687.

[9] Galileo laid the foundations of kinematics in *Discorsi e dimonstrazioni matematiche, intorno a due nuove scienze; attenenti alla mecanica i movimenti locali* published in 1638.

[10] The change of the attitude toward the notion of infinity can be seen by Nicolaus von Kues (1401–1464), see E. Knobloch, *Unendlichkeit und Mathematik bei Nicolaus von Kues* in A. Schürmann and B. Weiss, eds., *Chemie–Kultur–Geschichte, Festschrift für Hans-Werner Schütt* (Berlin: Verlag für Geschichte der Naturwissenschaften und der Technik, 2002), 223–4.

[11] A discussion of the theological background of modern science can be found in A. Funkenstein, *Theology and the Scientific Imagination* (Princeton: Princeton University Press, 1986) or in B. Gaal, *The Truth of Reason and the Reality of the World* (Debrecen: Debrecen Reformed College, 2002).