

RUSSELL, INFINITY, AND THE TRISTRAM SHANDY PARADOX

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INTRODUCTION

Mathematicians have puzzled for centuries what precisely we mean when we refer to the concept of *infinity*. Some have suggested that infinity is merely something that exists in the mind. Yet others maintain that infinity possesses some ontological status in the real world. In an attempt to demonstrate the difference between the *reality* of an infinite and the *idea* of an infinite, Aristotle had suggested the terms *actual infinite* (the completed whole value of infinity) and *potential infinite* (susceptible to infinite addition).⁽¹⁾ Analytic philosopher Bertrand Russell believed that an actual infinite could be achieved as long as the counter possessed an actually infinite number of years to do it. In the example given in Sterne's novel, we have the example of Tristram Shandy.⁽²⁾ Sterne writes about Tristram Shandy as an individual committed to writing an autobiography. However, he is so slow that it takes him one year in order to complete only one day. This means that the most recent event that could be recorded is the day that occurred one year ago. As Shandy writes an additional day, it takes him an additional year to complete the events of that day. Russell uses this example and believes that an actual infinite can be achieved through successive addition only if Shandy has an infinite number of days to complete it.

RUSSELL'S ASSESSMENT OF THE TRISTRAM SHANDY PARADOX

Bertrand Russell (1872-1970) suspects that the Tristram Shandy paradox can be solved. For Russell, it is the individual who possesses an infinite number of days. Of course mortal individuals possess merely a finite number of days. According to Russell, this is the key in solving the apparent problem. For a precise view of the problem, I will show the paradox numerically. The paradox posits an autobiographer who writes on every day passed. Since it takes Shandy one year (=365 days) to complete one day, then in terms of a one-to-one correspondence it would appear to be futile on a finite level:

Observed History: 1 day, 2 days, 3 days, 4 days, 5 days, . . .
Recorded History: 365 days, 730 days, 1095 days, 1460 days, 1825 days, . . .

It would seem mathematically impossible for Shandy to complete writing on all the days passed. Since each day yields an additional 365 days to write then It would seem that the longer Shandy wrote, the further behind he would get. Russell solves this mathematical problem by suggesting an actually infinite number of years in order to complete it. As a side note, the symbol often used to refer to a mathematical infinite is the Aleph Null (represented here as X_0 due to HTML limitations). As one observes the following equation, it appears to suggest something not true of usual, finite numbers.

$$X_0 + 1 = X_0$$

The implication here is that since any number added to infinity is still infinity, then the principle that *the whole is greater than the parts* does not apply here. One component of the equation (X0) is quantitatively equal to the sum of both components (X0 and 1). Russell asserts that given an infinite number of years to write plus the infinite number of days obtained results in an infinite amount of time transpired. Thus, the amount of time to write if obtained would be equal to the amount of time given to write about. Therefore (d = days to write on; y = years to complete; t = time obtained),

$$(y \times X0) + (d \times X0) = X0(t)$$

Russell believes that when the presence of infinity is seen all at once, then the concept of infinity is something that can exist as a quantitative property (he does not mean *exist* in the Platonic sense).⁽³⁾

WHAT AN ATHEIST AND A THEIST HAVE TO SAY ABOUT INFINITY

Throughout the years, mathematicians and philosophers have debated the notion of infinity as either a true property or just as a mathematical postulation non-representative of real numbers. At this time I will briefly present two authors who comment on Russell's assessment of the Tristram Shandy paradox.

Atheist Quentin Smith. Smith presents the Tristram Shandy paradox in his essay on the existence of infinity in the past. His purpose is to show that it is feasible to exist in a universe that does not have a beginning (this is in contrast to the theistic belief that the universe began to exist). Smith, with regard to Russell, makes this observation:

the number of past days written about is a proper subset of the infinite set of past days, and a proper subset of an infinite set can be numerically equivalent to the set even though there are members of the set that are not members of the proper subset. Just as the infinite set of natural numbers has the same number of members as its proper subset of equal numbers, yet has members that are not members of this proper subset (these members being the odd numbers); so the infinite set of past days has the same number of members as its proper subset of days written about, yet has members that are not members of this proper subset (these members being the days unwritten about).⁽⁴⁾

Smith maintains that the Tristram Shandy paradox is internally consistent in the light of sets and proper subsets.⁽⁵⁾ If it is true that

$$A = [1, 2, 3, 4, 5, \dots X0]$$

$$A1 = [2, 4, 6, 8, \dots X0]$$

$$A2 = [1, 3, 5, 7, \dots X0],$$

then:

- 1) Total numbers of proper subset A1 = Total numbers of set A.⁽⁶⁾
- 2) But, proper subset A1 does not contain the numbers of proper subset A2.
- 3) Therefore, proper subset A1 contains the same numbers as set A while *at the same time* not possessing numbers found in proper subset A2.

Smith shows that set theory validates the Tristram Shandy story and shows that the completion of his autobiography is logically possible.

Theist William Lane Craig. Craig, in response to Smith, asserts that both Russell and Smith have missed the real issue involved in the paradox. Craig responds to Russell that the problem with this argument seems to be that while an infinite number of years is a *necessary* condition of recording an infinite number of days at the rate of one day per year, it is not a *sufficient* condition. What is also needed is that the days and years be arranged in a certain way such that every day is succeeded by a year in which to record it. But then it will be seen that Tristram Shandy's task is inherently paradoxical; the absurdity lies not in the infinity of the past but in the task itself.⁽⁷⁾

Craig continues and claims that instead of Shandy writing forever and catching up on history, he would eventually be infinitely far behind.⁽⁸⁾ Craig further points out that the picture Russell paints entails a beginningless task. That is, if one were to ask "Where in the temporal series of events are the days recorded by Tristram Shandy at any given point?"⁽⁹⁾ then, according to Craig, one could only answer that the days are infinitely distant from the present. It appears that for every day Shandy is writing, there is an infinite distance from that day to the last recorded day. Craig shares with his critics that Russell's assessment fails to consider the real problem: *consecutive counting*.

ASSESSMENT

On the question of an actual infinite, my sympathies are with William Craig. I think both Russell and Smith have failed to convince me otherwise. At this point I will limit my response to Bertrand Russell's view of the Tristram Shandy paradox. My contention is that there seem to be two problems with Russell's view on this matter.

First, if we observe what is really going on in the paradox then it becomes apparent that it is logically unsolvable, not merely epistemologically unsolvable. When we see that for each day there are 365 subsequent days of writing, then I fail to see how Shandy "catches up" on the autobiography. Mere comparisons of infinite sets and subsets bespeak the paradox's lesson. It is precisely because an infinite proper subset equals an infinite set why successive addition fails to obtain. It appears, then, that we are discussing two different worlds: the finite and the infinite.⁽¹⁰⁾ So, if we uphold the principle of correspondence then it would appear that Tristram Shandy falls behind with each additional day he must write on.

Second, if we grant Russell's solution by granting immortality to Shandy, then it would appear that the problem is merely extended instead of solved. In other words, there is no reasons to adhere to an actual infinite by simply pushing back the length of the task to the time of the task. Aren't we still dealing with successive addition to the infinite? The question now becomes, "Will temporal segments of duration through successive addition arrive at an actual infinite?"⁽¹¹⁾ At this point it would be absurd to respond by suggesting that such an achievement is based on *finite time* because the argument now concerns time itself. Therefore, it would be question-begging to merely extend the problem.

CONCLUSION

We have seen Bertrand Russell attempt to prove the possibility of achieving an actual infinite through successive addition. Even though the Tristram Shandy paradox of the slow autobiographer was designed to show why such a successive addition is not possible, Russell believed that the solution required Shandy to have an infinite number of days to complete his task. Quentin Smith agreed with Russell's contention and suggested the notion of sets and proper subsets to prove the point. William Craig suggested that Russell had focused on the wrong issue and that the problem rested not in the necessary time to complete it but, rather,

on the sufficiency of consecutive counting. I concluded with two reasons why Russell had failed to solve the puzzle. I had suggested that Russell's solution actually increased the problem and then extended it to successive segments of time.

END NOTES

1. See Aristotle, *Physics*, 3.5.204b1-206a8. Some have charged that Aristotle was contradicting himself by asserting the fact that potentiality entails the idea that the actual can eventually be achieved. However, Aristotle's definition denotes a magnitude capable of being indefinitely divided or extended, not that the infinite would be achieved.
2. This example is cited in Abraham Fraenkel, *Abstract Set Theory* (Amsterdam: North-Holland Publishing Company, 1961), p. 6.
3. See Bertrand Russell, *The Principles of Mathematics*, 2nd ed. (London: Allen & Unwin, 1937), pp. 358-9.
4. Quentin Smith, "Infinity and the Past," *Theism, Atheism, and Big Bang Cosmology*, William L. Craig and Quentin Smith, eds. (Walton Street, Oxford: Oxford University Press, 1993), pp. 87-8.
5. For more information about sets and set theory, see Charles C. Pinter, *Set Theory* (Reading, Mass.: Addison-Wesley, 1971), p. 138.
6. The quantity of numbers in a set, subset, or proper subset is called the *cardinal number*.
7. William Lane Craig, "Time and Infinity," *ibid.*, p. 100.
8. *Ibid.*, p. 101.
9. *Ibid.*
10. It would appear to me that the failure of an actual infinite to succumb to the basic principles of mathematics is why the infinite remains forever untouched by finite beginning points.
11. Cf. *ibid.*, pp. 104-5.