Does Mathematical Beauty Pose Problems for Naturalism?

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The theme of this conference is Making All Things New: The Good, the True, and the Beautiful in the 21st Century. This paper will focus on features of truth and beauty contained in mathematics. More precisely, it asks whether aspects of mathematical theorizing, based mostly on notions of beauty and symmetry, and the subsequent success of mathematics in the natural sciences, cause difficulties for a naturalistic worldview. Several thinkers have raised these issues, at least indirectly, though not so much from the standpoint of mathematical beauty. We begin by reviewing some of their contributions.

In 1960 the physicist Eugene Wigner published "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" in Communications in Pure and Applied Mathematics. He begins his paper with a story about two friends talking about their jobs. One of them, a statistician, was working on population trends. He showed a paper to his friend. It started, as usual, with the Gaussian distribution, and the statistician explained the meaning of the symbols. His friend was a bit incredulous and was not quite sure whether the statistician was pulling his leg. "How can you know that?" was his query. "And what is this symbol here?"
"Oh," said the statistician, "this is pi."
"What is that?"
"The ratio of the circumference of a circle to its diameter."
"Well, now you are pushing your joke too far. Surely the population has nothing to do with the circumference of the circle."
Wigner uses that story to introduce two issues: (1) the surprising phenomenon that we have used mathematics so often to build successful theories; (2) the nagging question, "How do we know that, if we made a theory which focuses its attention on phenomena we disregard and disregards some of the

phenomena now commanding our attention, that we could not build another theory which has little in common with the present one but which, nevertheless, explains just as many phenomena as the present theory?"

Regarding Wigner's first point, he concedes that much of mathematics, such as Euclidean Geometry, has been developed because its axioms were modeled on what appeared to be true of the world. But this is not true for all—in fact most—of higher mathematics. Take my field, complex analysis, as just one example. It deals with "imaginary numbers"—things like the square root of minus one. In the 1500's such notions seemed odd to mathematicians because even negative numbers at that time were treated with some suspicion. There simply did not seem to be any physical reality corresponding to them, let alone their square roots. But that didn't stop mathematicians from using their imagination and pressing forward. The process that began the acceptance of complex numbers can legitimately be placed in the mid-fourteenth century when Scipione del Ferro of Bologna, and then later Niccolo Fontana solved the depressed cubic equation, which was later extended by Girolamo Cardano to the solution of the general cubic equation. Real-valued solutions to some cubic equations were then obtained by using these methods, but their solutions only came by using complex numbers as an intermediate step. The story that details the entire development of complex numbers is quite intricate, and it wasn't until the end of the 19th century that complex numbers became firmly entrenched. It is important to note, however, that complex numbers were studied because they were useful for mathematical and not physical purposes.

But complex numbers now play a pivotal role in helping physicists understand the quantum world. According to Wigner,

Quantum mechanics originated when Max Born noticed that some rules of computation, given by Heisenberg, were formally identical with the rules of computation with matrices. … Born, Jordan, and Heisenberg then proposed to replace by matrices the position and momentum variables of the equations of classical mechanics. … The results were quite satisfactory. However, there was … no rational evidence that their matrix mechanics would prove correct under more realistic conditions. As a matter of fact, the first application of their mechanics to a realistic problem, that of the hydrogen atom, was given several months later, by Pauli. This application gave results in agreement with experience. This was … understandable because Heisenberg's rules of calculation were abstracted from problems which included the old theory of the hydrogen atom. The miracle occurred only when matrix mechanics … was applied to problems for which Heisenberg's calculating rules were meaningless. Heisenberg's rules presupposed that the classical equations of motion had solutions with certain periodicity properties; and the equations of motion of the two electrons of the helium atom, or of the even greater number of electrons of heavier atoms, simply do not have these properties, so that Heisenberg's rules cannot be applied to these cases. Nevertheless, the calculation of the lowest energy level of helium, … [agreed] with the experimental data within the accuracy of the observations, which is one part in ten million.

"Surely," Wigner concludes, "in this case we 'got something out' of the equations that we did not put in."

I will not elaborate on the detail with which Wigner cites other examples including: Newton's law of motion—formulated in terms that appear simple to mathematicians, but which proved to be accurate beyond all reasonable expectations; quantum electrodynamics; or the theory of the Lamb shift—a purely mathematical theory.

Wigner ends his paper with the remarks,

… The enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious, and … there is no rational explanation for it. … The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure, even though perhaps also to our bafflement, to wide branches of learning.

In 1980 R.W. (Richard Wesley) Hamming took up the effectiveness issue raised by Wigner, and offered four tentative explanations that account for the applicability of mathematics. Let me review them briefly.

First, we see what we look for. Mathematicians craft postulates so that they will produce theories that conform to their prior observations. The Pythagorean theorem, Hamming claims, drove the formation of the geometric postulates and not vice versa.

Second, we select the kind of mathematics to use. By this Hamming simply means that we select the mathematics to fit the situation. The same mathematics simply does not work everywhere. Because we force mathematics onto particular situations, it is not all that surprising that we subsequently find it applicable.

Hamming's third comment is that science in fact answers comparatively few problems. To the extent that this is true, the less of a miracle the success of mathematics would appear to be. Wigner, as a physicist, certainly lived with mathematics as an indispensable tool. But other sciences do not share their reliance on mathematics, at least to the extent that physics does. Biology, it is said, has not been successfully dissected by the mathematical scalpel.

It seems to me, parenthetically, that this position is not obviously correct. A great deal of mathematical effort has been focused as of late on biological questions. A colleague of mine, for example, is currently looking at a Coxeter Groups as a model for DNA similarity. Also, knot theory, a newer branch of mathematics that deals with topological invariants, has had some success in the classification of DNA strands according to how they crinkle up under certain conditions. Even if we grant the argument, however, the success of mathematics in physics is something that cannot simply be dismissed by pointing to slower progress in other areas.

Finally, Hamming posits that the evolution of man provided the model, meaning the model for why humans are able to mathematize the physical universe. This is an interesting claim, but is not fleshed out beyond Hamming's remark that, "Darwinian evolution would naturally select for survival those competing forms of life which had the best models of reality in their minds—'best' meaning best for surviving and propagating." It is interesting to note that Hamming concludes with,

If you recall that modern science is only about 400 years old, and that there have been from 3 to 5 generations per century, then there have been at most 20 generations since Newton and Galileo. If you pick 4,000 years for the age of science, generally, then you get an upper bound of 200 generations. Considering the effects of evolution we are looking for via selection of small chance variations, it does not seem to me that evolution can explain more than a small part of the unreasonable effectiveness of mathematics.

I do not find this refutation compelling. Just as an inclined block needs a critical slope to overcome its friction and begin sliding, and once the sliding starts it proceeds rather rapidly, so too one might argue that, once science started it progressed quickly, but the evolutionary development that occurred before this explosion cannot be discounted.

But evolutionary accounts have problems as well. Let's briefly look at three such explanations. The first can be called the sexual selection hypothesis as argued by Geoffrey Miller. He claims that excessive capacities or acquisition of resources of any kind is basically a sexual display. If you've got the energy or time or intrinsic capacity to do things that don't have direct adaptive value—carrying around a set of antlers that are so big they are more of a detriment than a defense, or a peacock walking around with a big colored tail, or possessing artistic or mathematical brains that don't contribute to reproductive success—then that energy or time or intrinsic capacity by itself attracts mates.

Of course, physical attributes may well have some role in mate attraction, and artistic brains may as well insofar as they enable people to make attractive artifacts for display. The argument for mathematical brains, however, does not seem to hold up as well. Miller has some ways of dealing with this problem. For example, he states, "The healthy brain theory suggests that our brains are different from those of other apes not because extravagantly large brains helped us to survive or to raise offspring, but because such brains are simply better advertisements of how good our genes are. The more complicated the brain, the easier it is to mess up." But how would a larger brain be evident, and how would one somehow deduce that this is evidence of good genes? Such speculation seems to be forcing a theory when there may be no good evidence to support it.

Next is what we might call the module approach as argued by Stephen Mithin. Mithin writes from the perspective of an anthropologist, and has an enormous amount of archaeological data on which to draw. His thinking is that integrative and higher level (meta) cognitive processes grew out of the unification of specific evolutionary modules such as a module for tool use, or a module for interpersonal relations. He further argues that only in humans do we find a structure on top of modules—call it general purpose rationality.

This last approach has been extensively debated. For example, Alvin Plantinga's "Evolutionary Argument against Naturalism" claims that rationality is very unlikely a quality produced by survivability. Plantinga's approach, as he himself acknowledges, is similar to that found in C.S. Lewis's Miracles. Lewis's argument, incidentally, was recently enhanced by Victor Reppert in his book C.S. Lewis's Dangerous Idea. The thrust of Lewis's and Reppert's thinking is that you cannot get rationality out of a causally closed system that works solely on the basis of physical interactions operating in accordance with the laws of nature.

This brings us to the byproduct hypothesis, as exemplified by Pascal Boyer, who argues against Lewis's

and Reppert's view. His main thesis is that many higher cognitive functions (mathematics, art, religion, ethics, etc.) are not evolutionary adaptations at all. Instead, they are byproducts of things that are adaptive, and just piggyback on the adaptiveness of these other capacities. Some form of mathematical or quantitative ability is adaptive, Boyer argues, and as a byproduct of this we get the capacity to do higher order mathematics, the naked capacity of which at the time of its development wouldn't have been adaptive (or evolution wouldn't have known it was adaptive), though it may have turned out to have been adaptive. But I can't find any compelling evidence that would support Boyer in his contention, try as he might to produce one. His claim reminds me of scaffolding theories that are used to refute "Intelligent Design" arguments. If one is going to argue against something using an evolutionary framework, it behooves that person to supply a detailed model or story that will support such a refutation. Otherwise, the "God of the gaps" charge normally levied against design theorists can be turned around into, if you will, a "natural selection of the gaps" counter charge against the person arguing for blind chance natural selection.
Perhaps, though, evolutionary theory will eventually come up with a plausible explanation of our rationality. If so, any such theory that also attempts to promote a naturalistic world view would still run up against the arguments of Mark Steiner, author of The Applicability of Mathematics as a Philosophical Problem. Strictly speaking, Steiner's argument attempts to refute "Anthropocentrism" rather than Naturalism. But if Steiner is correct the naturalist should not take comfort. As far as I can tell, and Steiner shares this opinion, any form of Naturalism is defacto non-anthropocentric in that it would disallow a privileged status for humans in the scope of the universe. If, as Steiner argues, the success of mathematics can be shown to put humans in such a position, then naturalism has problems.

And just how does the success of mathematics put humans in a privileged position? For Steiner, it is not so much the success of any one particular mathematical theory in an area of science. After all, there have been many, many failures of mathematics in addition to its successes, and in this respect Steiner agrees with Hamming's third point and is thus critical of Wigner's approach in citing specific success examples from physics while ignoring error stories. The use of pi by the statistician in Wigner's opening line ignores all the failures, for example, in attempting to predict population trends. What Steiner is talking about is the success of mathematics as a grand strategy. It is a strategy that takes, for example, the raw formalisms of complex Hilbert space theory and boldly uses them as tools to make predictions about the quantum world, predictions that subsequently seem to be born out via experiment. And how is this phenomenon anthropocentric? Let me give an analogy. Most cultures use a base ten number system. No one is 100 per cent sure why this is the case, but the general consensus is that it has to do with our having 10 fingers. (Some primitive cultures use base 20, and to many this confirms the appendage hypothesis.) Now, what if successful theories of how the universe operates were based on multiples of 10? That would be anthropocentric in an extreme, as the only reason the number 10 is special to us is due to how we appear to ourselves.

Now suppose that, not only did the number 10 have special significance, but time and time again human aesthetic criteria played a significant role in understanding the universe. Such occurrences, when looked at from a meta-level, would surely make one wonder why such privilege seems to fall on the human species. Yet this situation is precisely analogous to what mathematicians and scientists actually do when they rely on human notions of beauty and symmetry in the development of their theories.

In fact, such activity has been a long standing and consistent strategy. Galileo, for example, pursued this tactic even though the best empirical evidence at the time did not support—indeed, it tended to disconfirm—his heliocentric theory. He adopted it because it seemed much more elegant than the Ptolemaic model. Most physicists generally admit that elegance, beauty, and symmetry hold primary sway in theory development. As Brian Green says in The Elegant Universe, “Physicists, as we have discussed, tend to elevate symmetry principles to a place of prominence by putting them squarely on the pedestal of explanation.” G. H. Hardy argues that mathematics itself, at least what constitutes good mathematics, is driven primarily by aesthetic criteria such as economy of expression, depth, unexpectedness, and seriousness, qualities that also seem to form standards for good poetry. The theories in mathematics that Hardy deems “important” are precisely the ones that satisfy these standards.

Regarding aesthetics, Steiner’s book contains several examples of beautiful mathematical systems being used in applications to the physical world, including the use of complex analysis in fluid dynamics, relativistic field theory, and thermodynamics. Let’s quickly examine two additional instances.

First, consider Schroedinger’s use of the wave equation. He begins with the equation , where he makes an assumption that energy is constant so he can eliminate it by differentiating. After a series of manipulations he gets , and then successfully uses his solution in situations where Energy is not constant. As Steiner says, this is “a perfect example of allowing the notation to lead us by the nose.”

In the interest of time, I will skip Steiner’s powerful examples from Quantum Mechanics (already alluded to when citing Wigner), where the tinkering of raw mathematical formalisms has often led to predictions about the quantum world that have subsequently panned out. Instead, I will focus on what at least one person considers to be a weak argument of Steiner’s: Maxwell’s anticipation of a physical reality. As you may know, Maxwell noted that the experimentally confirmed laws of Faraday, Coulomb, and Ampere, when put in differential form, contradicted the conservation of electrical charge. By working with Ampere’s law and adding a term to it, Maxwell got the laws to be consistent with, and indeed to imply, the conservation of charge. With no other warrant, Maxwell made the indicated changes and baldly predicted that his new term corresponded to some physical phenomenon. Ten years after his death Heinrich Hertz demonstrated the reality corresponding to this term—electromagnetic radiation.

Richard Carrier, a freelance writer who received his M.Phil. in Ancient History from Columbia University, is unimpressed by this episode, saying that what Maxwell did is entirely consistent with Naturalism. First, Maxwell’s putting laws in differential form conforms to the naturalistic observation that nature works in continuous, not broken, processes. Second, Maxwell took a logically sound hypothetical step: if charge isn’t being conserved, then it must be going somewhere. Carrier then states, “Maxwell rightly picked the simplest imaginable solution first, which due to human limitation is always the best place to start an investigation, and which statistically is the most likely [as] simple patterns and behaviors happen far more often than complex ones. [Thus] Maxwell's moves [that] anticipated EM radiation [were] therefore a natural conclusion from entirely naturalistic assumptions."

But with such language Carrier plays into Steiner's hands. Picking a simple solution in accordance with human limitations is precisely analogous to using the number 10 as a means of unlocking secrets to the universe. It is anthropocentrism in the extreme. I wonder, therefore, if it is difficult for people who were not trained in science to appreciate how absolutely uncanny is the continued use of mathematical formalisms by physicists. Green thus agrees with Steiner's main point: at least unconsciously, physicists have abandoned a raw naturalism in favor of a theory formation method that has principles of beauty imbedded in its core. If they are correct, this approach certainly appears to be an anthropocentric—and thus non-naturalistic—strategy.

Or could it be naturalistic after all? Might it not be argued that plausible evolutionary models can be devised that would explain, for example, the human preference for symmetry? I certainly think such constructs are likely, especially considering symmetries that might be adduced in examining our DNA code. But even if evolutionary thinking can explain our preference for symmetry, how can a "blind chance" form of such thinking explain why such preferences are successful?

Three strategies seem possible at this point. The first is to argue for some kind of probabilistic "weighting" that would drive physical processes towards the production of sentient life forms, and in such a way that their preferences for beauty coincide with the actual mechanisms of the universe. The second involves reverting to some kind of a primal basic position: it just so happens that the universe evolved in such a way that our notions for beauty happen to work. Finally, a thoroughgoing Postmodernist might argue (along the lines of Wigner's second question) that what we call successes came only because humans have invested a great deal of energy into science over the last 500 years. Who is to say that, if similar energies had been funneled in a different direction, there would be operating today a totally different paradigm yet with the same degree of "success"? The success is due to effort, not necessarily some amazing connection humans have with reality. Thus, mathematical beauty poses no problems for Naturalism whatsoever.

Time is running short, so let me give three very quick responses. First, with respect to the probabilistic weighting hypothesis, I wonder where the evidence is for this weighting. As Keith Ward comments, "A physical weighting ought to be physically detectable, … and it has certainly not been detected … In this sense, a continuing causal activity of God seems the best explanation of the progress towards greater consciousness and intentionality that one sees in the actual course of the evolution of life on earth." Next, although primal basic explanations are needed at some level, invoking them in an effort to explain the apparent privileged status for humans in the universe—they just do—appears akin to pulling a rabbit out of a hat. Likewise, the claim that our constructs of success are ad hoc appears to be an objection without any realistic alternative suggestion. It's almost like saying, "Well, your theory makes sense, but only if one buys into some of your commonly accepted cultural notions. Other—unspecified—theories will be able to show that the success you claim is really arbitrary, and thus not privileged."

I would suggest, in summary, that a theistic explanation here is the more plausible one in accounting for the continuing successes of mathematical theories that ultimately grow out of aesthetic criteria. In assessing these arguments it is hoped my listeners will adopt an approach similar to Reppert's in his defense of C.S. Lewis: There are, of course, valid points to be made on the side opposing these ideas, which should be looked at not as final answers, but as a spur to think through the relevant issues. It seems to me that human aesthetic values, and their subsequent use in successful physical theories, dovetail nicely with a Christian view that we are created in the image of God. Whatever being in God's image exactly entails, it seems to include a rational capacity reflective of his that enables humans to understand and admire his creation. While not a final answer, such a perspective seems to me very plausible, and one that a thinking person can confidently put in the marketplace of ideas for appraisal.